# Pharmaceutical statistics

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Dr. Rola Aqeel

ARKAN

ACADEMY

## (Paired Comparisons: Paired t-test)

- Previously we discussed the difference between two population means assuming that the samples were independent.
- Sometimes we may want to assess the effectiveness of a treatment or experimental procedure making use of observations resulting from dependent samples.
  - ➢ A hypothesis test based on this type of data is called paired comparison test.
  - ➢ Instead of performing the analysis with individual observations, we use $d_i$ (the difference between pairs of observations as the variable of interest).
  - ➢ When the n sample differences computed from the n pairs of measurements constitute a simple random sample from a normally distributed population of differences, the test statistic for testing hypothesis about the population mean difference $\mu_d$ is:

✓ $$t = \frac{\bar{d} - \mu_{d_0}}{SE}$$

where :

$\bar{d}$ is the sample mean difference

$\mu_{d_0}$ is the hypothesized population mean difference

$$SE = \frac{s_d}{\sqrt{n}}$$

n is the number of sample differences

$s_d$ is the standard deviation of the sample differences

★ Example:

In a study to evaluate the effect of very low calorie diet (VLCD) on the weight of 9 subjects, the following data was collected:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| B (before) | 117.3 | 111.4 | 98.6 | 104.3 | 105.4 | 100.4 | 81.7 | 89.5 | 78.2 |
| A (After) | 83.3 | 85.9 | 75.8 | 82.9 | 82.3 | 77.7 | 62.7 | 69 | 63.9 |

- ✓ The researchers wish to know if these data provide sufficient evidence to allow them to conclude that the treatment is effective in causing weight reduction in those individuals.
- ✓ If we choose ($d_i$ =A – B), the differences are: -34, -25.5, -22.8, -21.4, -23.1, -22.7, -19, -20.5, -14.3.
- ✓ Assumptions: the observed differences constitute a simple random sample from a normally distributed population of differences that could be generated.
- ✓ We may obtain the differences in one of two ways: by subtracting the before weights from the after weights (A – B) or by subtracting the after weights from the before weights (B – A).
- ✓ If the test is two sided and the question of interest is (is there a difference in mean body weight): A-B or B-A can be used $H_0$ and $H_a$ are the same for either:
- ✓ t- critical = $\pm$ t (1-$\alpha$/2, df = n-1)
- ✓ $H_0 : \mu_d = 0$
- ✓ $H_A : \mu_d \neq 0$

- ✓ If the question of interest is (does the VLCD result is significant weight reduction), $H_0$ and $H_A$ change on whether A-B or B-A is used as follows
- ✓

| A-B | B-A |
|---|---|
| $H_0$ $\mu_d \geq 0$, $H_a$ $\mu_d < 0$ | $H_0$ $\mu_d \leq 0$, $H_a$ $\mu_d > 0$ |
| t-critical = t ($\alpha$, df=n-1)<br>or t-critical = −t (1-$\alpha$, df= n-1) | t-critical = t (1-$\alpha$,n-1) |

- ✓ The test statistic: $t = \dfrac{\bar{d} - \mu_{d_0}}{s_{\bar{d}}}$
- ✓ Decision rule: Let $\alpha$=0.05, and the question of interest was (is their significant weight reduction after VLCD) (Based on A-B $H_0$: $\mu_d \geq 0$; $H_a$: $\mu_d < 0$, left sided) the critical value of $t_{\alpha,\ df=8}$ or $-t_{(1-\alpha,\ df=8)}$ is -1.86, reject $H_0$ if the computed t is less than or equal to the critical value.

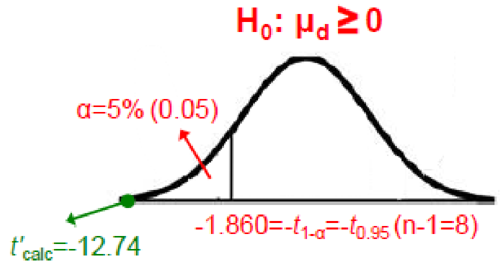
$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{-203.3}{9} = -22.5889$$

$$s_d^2 = \frac{\Sigma(d_i - \bar{d})^2}{n-1} = 28.2961$$

$$t = \frac{-22.5889 - 0}{\sqrt{\dfrac{28.2961}{9}}} = -12.7395$$

Reject $H_0$, since -12.7395 is in the rejection region.

We may conclude that the diet program is effective




- ➢ A 95% confidence interval for $\mu_d$ may be obtained as follows:
  - ✓ $\bar{d} \pm t_{(1-\alpha),df=8} * SE$

$$-22.5889 \pm 1.86\sqrt{28.2961/9}$$

$$-22.5889 \pm 4.0888$$

$$-26.68, -18.50$$

**t Table**

| cum. prob | $t_{.50}$ | $t_{.75}$ | $t_{.80}$ | $t_{.85}$ | $t_{.90}$ | $t_{.95}$ | $t_{.975}$ | $t_{.99}$ | $t_{.995}$ | $t_{.999}$ | $t_{.9995}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| one-tail | 0.50 | 0.25 | 0.20 | 0.15 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
| two-tails | 1.00 | 0.50 | 0.40 | 0.30 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.002 | 0.001 |
| df | | | | | | | | | | | |
| 1 | 0.000 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.31 | 636.62 |
| 2 | 0.000 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | 0.000 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | 0.000 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 0.000 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 0.000 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 0.000 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 0.000 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 0.000 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | 0.000 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |

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www.arkan-academy.com

+962 790408805